

A Systematic Approach to Confinement in $N = 1$ Supersymmetric Gauge Theories

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Abstract

We give necessary criteria for $N = 1$ supersymmetric theories to be in a smoothly confining phase without chiral symmetry breaking and with a dynamically generated superpotential. Using our general arguments we find all such confining SU and Sp theories with a single gauge group and no tree level superpotential.

Following the initial breakthrough in the works of Seiberg on exact results in $N = 1$ supersymmetric QCD (SQCD) [1], much progress has been made in extending these results to other theories with different gauge and matter fields [2-11]. We now have a whole zoo of examples of supersymmetric theories for which we know results about the vacuum structure and the infrared spectrum. A number of theories are known to have dual descriptions, others are known to confine with or without chiral symmetry breaking, and some theories do not possess a stable ground state.

Unfortunately, we are still lacking a systematic and general approach that allows one to determine the infrared properties of a given theory. The results in the literature have mostly been obtained by an ingenious guess of the infrared spectrum. This guess is then justified by performing a number of non-trivial consistency checks which include matching of the global anomalies, detailed study of the moduli space of vacua, and the behavior of the theory under perturbations.

In this letter, we will depart from the customary trial and error procedure and give some general arguments which allow us to classify a subset of supersymmetric theories. To be specific, we intend to answer the general question of which supersymmetric field theories may be confining without chiral symmetry breaking and with a confining superpotential. We present a few simple arguments which allow us to rule out most theories as possible candidates for confinement without chiral symmetry breaking. For the most part, these arguments already exist in the literature but our systematic way of putting them to use is new. As a demonstration of the power of our arguments we give a complete list of all $SU(N)$ and $Sp(N)$ gauge theories with no tree level superpotential which confine without chiral symmetry breaking, and we determine the confined degrees of freedom and the superpotential describing their interactions (“confining superpotential”).

To begin, let us first explain what we mean by “smooth confinement without chiral symmetry breaking and with a non-vanishing confining superpotential”, which, from now on, we will abbreviate by s-confinement. We will call a theory confining when its infrared physics can be described exactly in terms of gauge invariant composites and their interactions. This description has to be valid everywhere on the moduli space of vacua. Our definition of s-confinement also requires that the theory dynamically generates a confining superpotential, which excludes models of the type presented in Ref. [11]. Furthermore, the phrase “without chiral symmetry breaking” implies that the origin of the classical moduli space is also a vacuum in the quantum theory. In this vacuum, all the global symmetries of the ultraviolet remain unbroken. Finally, the confining superpotential is a holomorphic function of the confined degrees of freedom and couplings, which describes all the interactions in the extreme infrared. Note that this definition excludes theories which are in a Coulomb phase on a submanifold of the moduli space [2], or theories which have distinct Higgs and confining phases with associated phase boundaries on the moduli space.

Our prototype example for an s-confining theory is Seiberg’s SQCD [1] with the number of flavors F chosen to equal $N + 1$, where N is the number of colors, and a

“flavor” is a pair of matter fields in the fundamental and antifundamental representations of $SU(N)$. Seiberg argued that the matter fields Q and \bar{Q} are confined into “mesons” $M = Q\bar{Q}$ and “baryons” $B = Q^N$, $\bar{B} = \bar{Q}^N$. At the origin of moduli space all components of the mesons and baryons are massless and interact via the confining superpotential

$$W = \frac{1}{\Lambda^{2N-1}}(\det(M) - BM\bar{B}). \quad (1)$$

At this point, the full global $SU(N+1) \times SU(N+1) \times U(1)_R \times U(1)$ global symmetry of the model is unbroken, and it is a non-trivial consistency check that all global anomalies are matched by the mesons and baryons. The equations of motion $M^{-1}\det(M) - B\bar{B} = 0$, $M\bar{B} = 0$, and $BM = 0$, when expressed in terms of the original degrees of freedom, Q and \bar{Q} , are identical to the classical constraints. This constitutes another consistency check: the quantum theory should reproduce these constraints in the classical limit, $\Lambda \rightarrow 0$, or for generic large vacuum expectation values (VEVs) which completely break the gauge group.

Other examples in the literature for theories which s-confine include $SU(N)$ with an antisymmetric tensor, $N-4$ antifundamentals, and four flavors [3], $Sp(2N)$ with $2N+4$ fundamentals [4], $Sp(2N)$ with an antisymmetric tensor and six fundamentals [5, 6], a few $SO(N)$ theories with spinors, and G_2 with five fundamentals [8, 9].

We now present our arguments which enable us to identify other theories which s-confine. Except for the discussion of generalizations at the end of this letter we limit our attention to theories with one gauge group and vanishing tree level superpotential.

The first argument follows from the requirement of smoothness of the confining superpotential at the origin of moduli space. In the absence of a tree level superpotential and with only one gauge group, the global symmetries and holomorphy are sufficient to completely determine the form of any non-perturbatively generated superpotential [12]. For a theory with gauge group G and matter fields ϕ_i this superpotential is

$$W \propto \left(\prod_i \phi_i^{\mu_i} \right)^{2/(\sum_j \mu_j - \mu(G))}, \quad (2)$$

where $\mu(G)$ is the Dynkin index¹ of the adjoint representation of G , and μ_i are the indices of the representations of the ϕ_i . Note that there may be several (or zero) possible contractions of gauge indices, thus the superpotential can be a sum of several terms. We require the coefficient of this superpotential to be non-vanishing, then holomorphy at the origin implies that the exponents of all fields ϕ_i are positive

¹We normalize the index of the fundamental representation to 1.

integers. Therefore², $\sum_j \mu_j - \mu(G) = 1$ or 2 , and for SU and Sp theories anomaly cancellation further constrains

$$\sum_j \mu_j - \mu(G) = 2. \quad (3)$$

This formula constitutes a necessary condition for s-confinement, it enables us to rule out most theories immediately. For example, for SQCD we find that the only candidate theory is the theory with $F = N + 1$. Unfortunately, Eq. 3 is not a sufficient condition. An example for a theory which satisfies Eq. 3 but does not s-confine is $SU(N)$ with an adjoint superfield and one flavor. This theory is easily seen to be in an Abelian Coulomb phase for generic VEVs of the adjoint scalars and vanishing VEVs for the fundamentals.

We could now simply examine all theories that satisfy Eq. 3 by finding all independent gauge invariants and checking if this ansatz for the confining spectrum matches the anomalies. Apart from being very cumbersome, this method is also not very useful to demonstrate that a given theory satisfying Eq. 3 is not s-confining.

A better strategy relies on our second observation. An s-confining theory with a smooth description in terms of gauge invariants at the origin must also be s-confining everywhere on its moduli space. This is because the confining superpotential at the origin which is a simple polynomial in the fields is analytical everywhere, and no additional massless states are present anywhere on the moduli space. Therefore, the theory restricted to a particular flat direction must have a smooth description as well. This observation has two very useful applications.

First, if we have a theory that s-confines and we know its confined spectrum and superpotential, we can easily find new s-confining theories by going to different points on moduli space. In the ultraviolet description, the gauge group is broken to a subgroup of the original group, some matter fields are eaten by the Higgs mechanism, and the remaining ones decompose under the unbroken subgroup. The corresponding confined description is obtained by simply finding the corresponding point on the moduli space of the confined theory. The global symmetries will be broken in the same way, and some fields may be massive and can be integrated out. This newly found confined theory is guaranteed to pass all the standard consistency checks because they are a subset of the consistency checks for the original theory. For example, the anomalies of the new s-confining theory are guaranteed to match: the unbroken global symmetries are a subgroup of original global symmetries, and the anomalies under the subgroup are left unchanged – both in the infrared and ultraviolet descriptions – because the fermions which obtain masses give cancelling contributions to the anomalies.

²Other solutions exist if all μ_i have a common divisor d , then for $\sum_j \mu_j - \mu(G) = d$ or $2d$ the superpotential Eq. 3 may be regular. We will argue at the end of the paper that these solutions generically do not yield s-confining theories. Another possibility is that the coefficient of the superpotential above vanishes. We will consider this special case in our discussions at the end as well.

Second, the above observation can be turned around to provide another necessary condition for s-confinement. If anywhere on the moduli space of a given theory we find a theory which is not s-confining or completely higgsed, we know that the original theory cannot be s-confining either.

Let us study some examples. Suppose we knew that $SU(N)$ with $N + 1$ flavors for some large N is s-confining, then we could immediately conclude that the theories with $n < N$ also s-confine. We simply need to give a VEV to some of the quark-antiquark pairs to break $SU(N)$ to any $SU(n)$ subgroup. The quarks with vevs are eaten, leaving $n + 1$ flavors and some singlets. We remove these singlets by adding “mirror” superfields with opposite global charges and giving them a mass. We now identify the corresponding point on the moduli space of the confined $SU(N)$ theory. Some fields obtain masses from the superpotential of Eq. 1 when we expand around the new point in moduli space. After integrating the massive fields and removing the fields corresponding to the singlets in the ultraviolet theory via masses with mirror partners, we obtain the correct confined description of $SU(n)$.

A non-trivial example of a theory which can be shown to not s-confine is $SU(4)$ with three antisymmetric tensors and two flavors. This theory satisfies Eq. 3 and is therefore a candidate for s-confinement. By giving a VEV to an antisymmetric tensor we can flow from this theory to $Sp(4)$ with two antisymmetric tensors and four fundamentals. VEVs for the other antisymmetric tensors let us flow further to $SU(2)$ with eight fundamentals which is known to be at an interacting fixed point in the infrared. We conclude that the $SU(4)$ and $Sp(4)$ theories and all theories that flow to them cannot be s-confining either. This allows us to rule out the following chain of theories, all of which are gauge anomaly free and satisfy Eq. 3

$$\begin{array}{ccccccc}
SU(7) & \rightarrow & SU(6) & \rightarrow & SU(5) & \rightarrow & SU(4) & \rightarrow & Sp(4) \\
\begin{array}{c} \square \\ \square \end{array} 2\square 4\bar{\square} & & \begin{array}{c} \square \\ \square \end{array} \begin{array}{c} \square \\ \square \end{array} \square 3\bar{\square} & & 2\begin{array}{c} \square \\ \square \end{array} \begin{array}{c} \square \\ \square \end{array} \square 2\bar{\square} & & 3\begin{array}{c} \square \\ \square \end{array} 2\square 2\bar{\square} & & 2\begin{array}{c} \square \\ \square \end{array} 4\square
\end{array} \quad (4)$$

Note that a VEV for one of the quark flavors of the $SU(4)$ theory lets us flow to an $SU(3)$ theory with four flavors which is s-confining. We must therefore be careful, when we find a flow to an s-confining theory, it does not follow that the original theory is s-confining as well. The flow is only a necessary condition. However, we suspect that a theory with a single gauge group and no tree-level superpotential is s-confining if it is found to flow to s-confining theories in all directions of its moduli space. We do not know of any counter examples.

Armed with formula in Eq. 3 and our observation on flows of s-confining theories, we were able to find all s-confining SU and Sp gauge theories with a single gauge group and no tree-level superpotential for arbitrary tensor representations. To achieve this, we first found all possible matter contents satisfying Eq. 3. We list all these theories in Table 1. We then studied the possible flows of these theories and discarded all those with flows to theories which do not s-confine. This process eliminated all except about a dozen theories for which we then explicitly determined the independent gauge

invariants and matched anomalies to find the confining spectra. These results are summarized in Table 1.

Six of the ten theories which s-confine are new³: $SU(N)$ with $\square + \bar{\square} + 3\square + 3\bar{\square}$, $SU(7)$ with $2\square + 6\bar{\square}$, $SU(6)$ with $2\square + \square + 5\bar{\square}$, $SU(6)$ with $\square + 4\square + 4\bar{\square}$, $SU(5)$ with $2\square + 2\square + 4\bar{\square}$, and $SU(5)$ with $3\square + 3\bar{\square}$. For the theories which do not s-confine we indicated the method by which we obtained this result: either by noting that the theory has a branch with only unbroken $U(1)$ gauge groups, or else by flowing along a flat direction to a theory with smaller non-Abelian gauge group which does not s-confine.

Detailed results on the new theories including the confining spectra, superpotentials, various flows, and consistency checks will be reported elsewhere [14]. Here, we just point out a few salient features.

Most of the new s-confining theories contain vector-like matter. Perturbing these theories by adding mass terms for some of the vector-like matter, we easily obtain exact results on the theories with the matter integrated out. Among the theories that we find in this way are new theories which confine with chiral symmetry breaking, theories with runaway vacua, and theories which confine without chiral symmetry breaking and vanishing superpotentials. Since many of the new theories presented here are chiral, they can be used to find models of dynamical supersymmetry breaking along the lines of Refs. [15]. Examples for such supersymmetry breaking theories will also be included in the detailed paper [14]. Our s-confining theories might be used for building extensions of the standard model with composite quarks and leptons [16].

Finally, we comment on possible exceptions and generalizations of our arguments. A possible exception to our condition in Eq. 3 arises, when all μ_i and $\mu(G)$ have a common divisor. Then the superpotential Eq. 2 can be holomorphic even when $\sum_j \mu_j - \mu(G) \neq 2$. However, whereas Eq. 3 is preserved under most flows, the property that all μ 's have a common divisor is not. Therefore, such theories flow to theories which are not s-confining, and by our second necessary condition the original theory is not s-confining either.

Another possibility is that the confining superpotential vanishes, and the confined degrees of freedom are free in the infrared. This can only happen if there are no classical constraints among the basic gauge invariant operators which satisfy the 't Hooft anomaly matching conditions, otherwise the quantum solution would not have the correct classical limit. Examples of theories which are believed to confine in this way can be found in the literature [7, 11, 14].

Generalizations to $SO(N)$ groups are not completely straightforward because in the case of $SO(N)$ theories “exotic composites” containing the chiral superfield W_α might appear in the infrared spectrum and superpotential, thus modifying our argument and result of Eq. 3.

³Some of these results were obtained independently by A. Nelson [13].

$SU(N)$	$(N+1)(\square + \bar{\square})$	s-confining
$SU(N)$	$\square + N\bar{\square} + 4\square$	s-confining
$SU(N)$	$\square + \bar{\square} + 3(\square + \bar{\square})$	s-confining
$SU(N)$	Adj $+\square + \bar{\square}$	Coulomb branch
$SU(4)$	Adj $+\square$	Coulomb branch
$SU(4)$	$3\square + 2(\square + \bar{\square})$	$SU(2)$: $8\square$
$SU(4)$	$4\square + \square + \bar{\square}$	$SU(2)$: $\square\square + 4\square$
$SU(4)$	$5\square$	Coulomb branch
$SU(5)$	$3(\square + \bar{\square})$	s-confining
$SU(5)$	$2\square + 2\square + 4\bar{\square}$	s-confining
$SU(5)$	$2(\square + \bar{\square})$	$Sp(4)$: $3\square + 2\square$
$SU(5)$	$2\square + \bar{\square} + 2\bar{\square} + \square$	$SU(4)$: $3\square + 2(\square + \bar{\square})$
$SU(6)$	$2\square + 5\bar{\square} + \square$	s-confining
$SU(6)$	$2\square + \bar{\square} + 2\bar{\square}$	$SU(4)$: $3\square + 2(\square + \bar{\square})$
$SU(6)$	$\square + 4(\square + \bar{\square})$	s-confining
$SU(6)$	$\square + \bar{\square} + 3\bar{\square} + \square$	$SU(5)$: $2\square + \bar{\square} + 2\bar{\square} + \square$
$SU(6)$	$\square + \bar{\square} + \bar{\square}$	$Sp(6)$: $\square + \bar{\square} + \square$
$SU(6)$	$2\square + \square + \bar{\square}$	$SU(5)$: $2(\square + \bar{\square})$
$SU(7)$	$2(\square + 3\bar{\square})$	s-confining
$SU(7)$	$\square + 4\bar{\square} + 2\square$	$SU(6)$: $\square + \bar{\square} + 3\bar{\square} + \square$
$SU(7)$	$\square + \bar{\square} + \square$	$Sp(6)$: $\square + \bar{\square} + \square$
$Sp(2N)$	$(2N+4)\square$	s-confining
$Sp(2N)$	$\square + 6\square$	s-confining
$Sp(2N)$	$\square\square + 2\square$	Coulomb branch
$Sp(4)$	$2\square + 4\square$	$SU(2)$: $8\square$
$Sp(4)$	$3\square + 2\square$	$SU(2)$: $\square\square + 4\square$
$Sp(4)$	$4\square$	$SU(2)$: $2\square\square$
$Sp(6)$	$2\square + 2\square$	$Sp(4)$: $2\square + 4\square$
$Sp(6)$	$\square + 5\square$	$Sp(4)$: $2\square + 4\square$
$Sp(6)$	$\square + \bar{\square} + \square$	$SU(2)$: $\square\square + 4\square$
$Sp(6)$	$2\square$	$SU(3)$: $\square\square + \square\square$
$Sp(8)$	$2\square$	$Sp(4)$: $5\square$

Table 1: All SU and Sp theories satisfying $\sum_j \mu_j - \mu(G) = 2$. Note that this list is finite because the indices of higher index tensor representations grow very rapidly with the size of the gauge group. We list the gauge group and the field content of the theories in the first column. In the second column, we indicate which theories are s-confining. For the remaining ones we give the flows to non-confining theories or indicate that there is a Coulomb branch on the moduli space.

Generalizations to theories with more than one gauge group or tree level superpotentials are more difficult. The additional interactions break some of the global symmetries which are now not sufficient to completely determine the functional form of the confining superpotential. Another complication is that in these theories the flat directions of the quantum theory are sometimes difficult to identify. Since our second argument only applies to flows in directions which are on the quantum moduli space, incorrect conclusions would be obtained from flows along classical flat directions which are not flat in the quantum theory.

In summary, we have discussed general criteria for s-confinement and used them to find all s-confining theories with $SU(N)$ or $Sp(2N)$ gauge groups.

It is a pleasure to thank P. Cho, A. Cohen, N. Evans, L. Randall, and R. Sundrum for useful discussions. We also thank B. Dobrescu, A. Nelson, and J. Terning for comments on the manuscript. C.C. and W.S. are supported in part by the U.S. Department of Energy under cooperative agreement #DE-FC02-94ER40818. M.S. is supported by the U.S. Department of Energy under grant #DE-FG02-91ER40676.

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